SIMULATION OF RAYLEIGH BERNARD CONVECTION USING LATTICE BOLTZMANN METHOD

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SUPERVISOR'S DECLARATION

I hereby declare that I have checked this project and in my opinion, this project is adequate in terms of scope and quality for the award of the degree of Bachelor of Mechanical Engineering.

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STUDENT'S DECLARATION

I hereby declare that the work in this project is my own except for quotations and summaries which have been duly acknowledged. The project has not been accepted for any degree and is not concurrently submitted for award of other degree.

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Dedicated to my beloved parents

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ABSTRACT

In this thesis, a method of lattice Boltzmann is introduced. Lattice Boltzmann Method is to build a bridge between the microscopic and macroscopic dynamics, rather than to deal with macroscopic dynamics directly. In other words, LBM is to derive macroscopic equations from microscopic dynamics by means of statistic, rather than to solve macroscopic equations. Then, the methodology and general concepts of the lattice Boltzmann method are introduced. Next, a thermal lattice Boltzmann model is developed to simulate incompressible thermal flow. This report describes the flow pattern of Rayleigh Bernard Convection. This project will be focusing at low Rayleigh number and discretization of microscopic velocity using 9-discrete velocity model (D2Q9) and 4-discrete velocity model (D2Q4). This two discrete velocity model is applying the Gauss-Hermitte quadrate procedure. Rayleigh Bernard Convection and Lattice Boltzmann Method have been found to be an efficient and numerical approach to solve the natural convection heat transfer problem. Good Rayleigh Bernard Convection flow pattern agreement was obtained with benchmark (previous study).

ABSTRAK

Di dalam tesis ini, kaedah kekisi Boltzmann diperkenalkan. Kaedah kekisi Bolzmann ialah untuk membuat hubungan di antara mikroskopik dan makroskopik. Dalam erti kata lain, kaedah kekisi Boltzmann ialah menerbitkan persamaan makroskopik daripada pergerakan mikoskopik oleh statistik daripada menyelesaikan persamaan makroskopik. Kemudian, methodogi dan konsep umum kaedah kekisi Boltzmann diperkenalkan. Kemudian, model terma kekisi Boltzmann adalah untuk membina simulasi di dalam aliran terma yg tidak mampat. Laporan ini memperihalkan corak aliran arus perolakan Rayleigh Bernard. Projek ini menumpukan pada nombor Rayleigh yang rendah dan arah pergerakan halaju mikoskopik menggunakan model 9-diskrit halaju dan model 4-diskrit halaju. Kedua-dua model diskrit ini mengadaptasikan prosedur kuadrat Gauss-Hermitte. Pemanasan di antara dua plat dan kaedah kekisi Boltzmann merupakan satu cara yang berkesan untuk menyelesaikan pemanasan pemindahan haba. Bentuk aliran di antara dua plat mempunyai persamaan dengan bentuk aliran kajian sebelum.

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LIST OF SYMBOLS

u	Velocity of the fluid parcel
P	Pressure
ρ	Fluid density
$\tau_{f}\ g$	Time relaxation
υ	Viscosity of fluid
χ	Thermal diffusivity
$\Omega_{(x\ t)}$	Single relaxation time
$f(\mathbf{x},t)$	Current distribution of particles
$f^{eq}(\mathbf{x},t)$	Equilibrium function
Ω	Collision integral
Δt	Value of unity
T_C	Cool plate temperature
T_H	Hot plate temperature
α	Thermal diffusivity
k	Thermal conductivity
c_p	Specific heat
x	Characteristic length
Ra_x	Rayleigh number at position x

Pr	Prandtl number
g	Acceleration due to gravity
β	Thermal expansion coefficient
V	Mean fluid velocity
Q	Volumetric flow rate
A	Pipe cross-sectional area
d	Depth of the chamber
δ	Thickness
L	Length
Gr_x	Grashof number at position x

LIST OF ABBREVIATIONS

BGK	Bhatnagar Groos Krook
CFD	Computational fluid dynamics
D2Q4	Two dimensionals four velocities model
D2Q9	Two dimensionals nine velocities model
LB	Lattice Boltzmann
LBE	Lattice Boltzmann equation
LBM	Lattice Boltzmann method
LGA	Lattice gas approach
MD	Molecular dynamics

CHAPTER 1

INTRODUCTION

1.1 PROJECT BACKGROUND

Navier-Stokes Equation

$$\frac{\partial u}{\partial t} + u\nabla \bullet u = -\nabla P + \left(\frac{2\tau_f - 1}{6}\right)\nabla^2 u \qquad (1.1)$$

$$\nabla . u = 0 \qquad (1.2)$$

Equation 1.1 shows the Navier-Stokes Equation that explains the flow of incompressible fluids together with the continuity equation show in equation 1.2 where ν is the kinematics viscosity, $\mathbf{u}$ is the velocity of the fluid parcel, P is the pressure, and ρ is the fluid density.

Lattice Gas Approach

Lattice Boltzmann models were first based on Lattice Gas Approach (LGA) in that they used the same lattice and applied the same collision. Instead of particles, Lattice Boltzmann (LBM) deal with continuous distribution functions which interact locally (only distributions at a single node are involved) and which propagate after collision to the next neighbor node. This is the main advantage of LBM compare to LGA. The next step in the development was the simplification of the collision and the choice of different distribution functions. This gives LBM is more flexible than LGA.

Molecular Dynamics

In molecular dynamics (MD), one tries to simulate macroscopic behavior of real fluids by setting up the model which described the microscopic interaction as good as possible. This leads to realistic equation of states whereas LGA or LBM posses only isothermal relations between mass density and pressure. The complexity of the interactions in MD restricts the number of particles and the time of integration. A method somewhat in between MD and LGA is maximally discretized molecular dynamics proposed by Colvin, Ladd and Alder (1988).

Lattice Boltzmann Method

The lattice Boltzmann method (LBM) has developed into an alternative and promising numerical scheme for simulating fluid flows and modeling physics in fluids. Historically, the lattice Boltzmann approach developed from improvement of lattice gases, although it can also be derived directly from the simplified Bhatnagar-Gross-Krook (BGK) equation.

The lattice Boltzmann method is based on microscopic models and macroscopic kinetic equations. The kinetic nature of the LBM introduces important features that distinguish it from other numerical methods. First, the streaming process of the LBM in velocity space is linear. Second, the incompressible Navier-Stokes (NS) equations can be obtained in the nearly incompressible limit of the LBM. The LBM originated from lattice gas approach (LGA), a discrete particle kinetics utilizing a discrete lattice and discrete time. The primary goal of LBM is to build a bridge between the microscopic and macroscopic dynamics rather than to deal with macroscopic dynamics directly. In other words, the goal is to derive macroscopic equations from microscopic dynamics by means of statistics rather than to solve macroscopic equation in Figure 1.1 below.

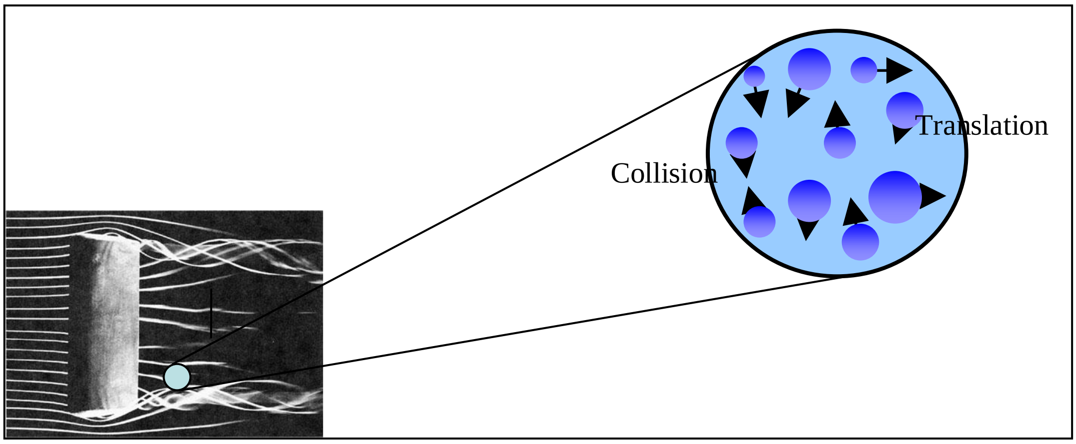

Figure 1.1: The relationship between macroscopic and microscopic.

Rayleigh-Bernard Convection

Rayleigh-Bernard convection is the instability of a fluid layer which is confined between two thermally conducting plates, and is heated from below to produce a fixed temperature difference. For small temperature differences between the plates there is no flow and heat is transported by conduction only. Above a certain temperature differences, convection sets in against the downward pointing gravitational acceleration, and a regular convection pattern is formed. At even higher temperature differences this pattern breaks down, eventually leading to plume dominated convection turbulence. Since liquids typically have positive thermal expansion coefficient, the hot liquid at the bottom of the cell expands and produces an unstable density gradient in the fluid layer. If the density gradient is sufficiently strong, the hot fluid will rise, causing a convective flow which results in enhanced transport of heat between the two plates.

1.2 PROBLEM STATEMENT

Simulate Rayleigh Bernard Convection using Lattice Boltzmann Method to study the flow pattern of Rayleigh Bernard Convection.

1.3 OBJECTIVE

For this thesis the objective is to study the flow pattern of Rayleigh Bernard Convection.

1.4 SCOPE OF WORK

This project is focusing on the pattern of the Rayleigh Bernard Convection flow pattern at low Rayleigh number which is at laminar flow. This project also focusing on using D2Q9 and D2Q4 microscopic velocity model for discretization of microscopic velocity.

1.5 PROCESS FLOW CHART

The Figure 1.2 shows the separation of information or processes in a step-by-step flow and easy to understand diagrams showing how steps in a process fit together. This makes useful tools for communicating how processes work and for clarity due to time limitation how a particular job is done in Final Year Project.

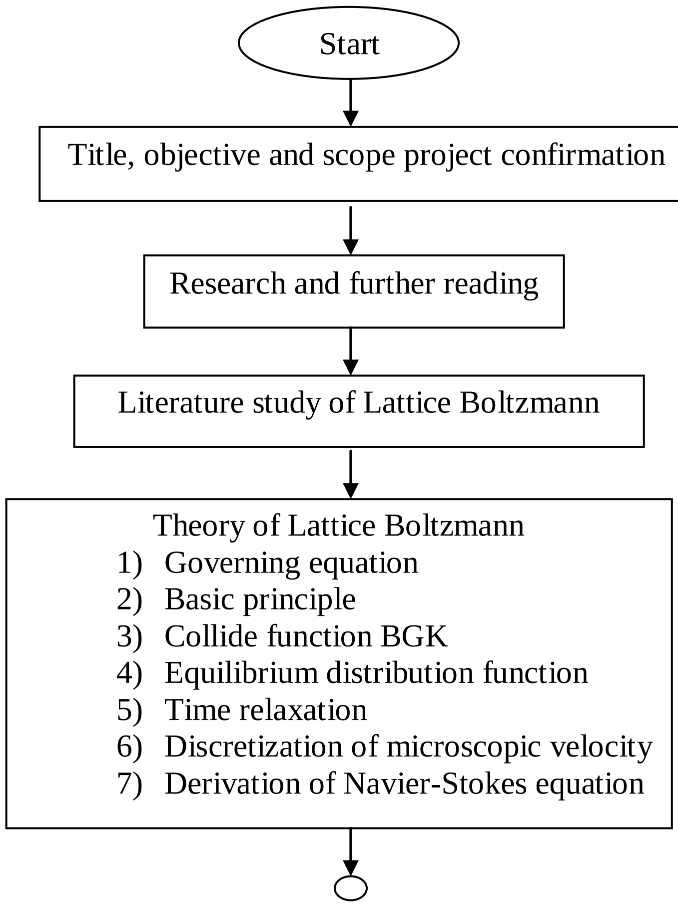

Figure 1.2: Flowchart of PSM

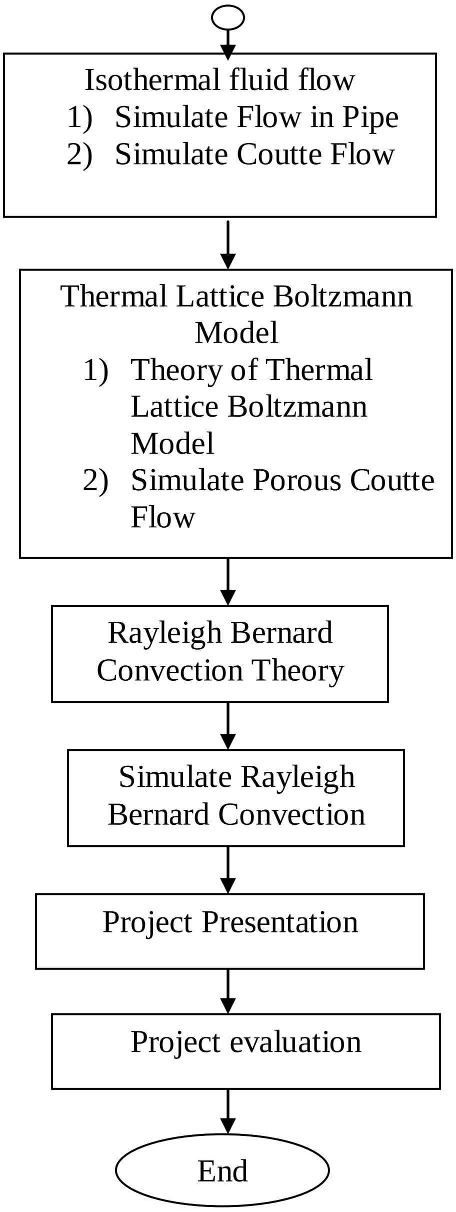

Figure 1.2: continue

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter, the previous study for Bhatnagar–Gross-Krook (BGK) collision equation and model, Lattice Boltzmann Equation (LBE) will be discussed in this chapter. From BGK and LBE, discretization of microscopic velocity will be discussed. Discretization of microscopic velocity consist two models which are 9-discrete velocity model (D2Q9) and are 4-discrete velocity model (D2Q4). Furthermore, the dimensionless number (Prandlt number, Rayleigh number, Reynolds number and Nusselt number) also will be discussed in this chapter. Lastly, Rayleigh Bernard Convection and Rayleigh Bernard Convection problem will be discussed in this chapter. Rayleigh Bernard Convection problem is the boundary condition that will be used for the simulation.

2.2 NAVIER-STOKES EQUATION

$$\rho \frac{Du}{Dt} = -\nabla p + \rho f \tag{2.1}$$

$$\nabla . u = 0 \tag{2.2}$$

Sources: S. Chen and G.D. Doolen, Annu 1998.

Equation 2.1 shows the fluid velocity u of an inviscid (ideal) fluid of density ρ under the action of a body force ρ f is determined and equation 2.2 shows as Euler's equation. The scalar p is the pressure. This equation is supplemented by an equation describing the conservation of mass. For an incompressible fluid this is simply to get the continuity equation.

Real fluids however are never truly inviscid. We must therefore see how Euler's equation is changed by the inclusion of viscous forces.

$$\frac{\partial u}{\partial t} + u\nabla \bullet u = -\nabla P + \left(\frac{2\tau_f - 1}{6} \right) \nabla^2 u \qquad (2.3)$$

Sources: C. S. Nor Azwadi 2007.

Equation 2.3 shows the flow of incompressible fluids can be described by the momentum equation. Derivation of continuity and momentum equation, we will get the Navier-Stokes equation by using Chapmann-Enskog expansion procedure. Chapmann-Enskog procedure is a method for obtaining an approximate solution. Chapmann-Enskog also manages to extract from the kinetic equation for the density distribution function, *F* a set of hydrodynamic equations for the particle number, the momentum and the energy per unit volume.

2.2.1 Macroscopic Equation for Isothermal

$$\upsilon = \frac{2\tau - 1}{6} \qquad (2.4)$$

Sources:M. Rohde, D. Kandhai, J. J. Derksen, and H. E. A. Van den Akker 2003.

Using Chapmann-Enskog expansion procedure, relation between the time relaxation τ , in microscopic level and viscosity of fluid υ, in macroscopic level is shown above.

2.2.2 Macroscopic Equation for Thermal

$$\frac{\partial T}{\partial t} + \nabla \cdot (\mathbf{u}T) = \left(\tau_g - \frac{1}{2} \right) \nabla^2 T \tag{2.5}$$

Sources: C. S. Nor Azwadi 2007.

In thermal model, the energy equation is added. The energy equation is shown above.

$$\tau_f = 3\upsilon + \frac{1}{2} \tag{2.6}$$

$$\tau_g = \chi + \frac{1}{2} \tag{2.7}$$

M. Rohde, D. Kandhai, J. J. Derksen, and H. E. A. Van den Akker 2003.

Viscosity and thermal diffusivity will produce when using the Chapmann-Enskog expansion procedure where υ is viscosity and χ is thermal diffusivity show in equation 2.6 and equation 2.7.

2.3 BHATNAGAR-GROSS-KROOK (BGK)

The integral-differential Boltzmann equation (proposed by Boltzmann in 1872) can be solving by the model of Bhatnagar–Gross-Krook that had been proposed in 1954. The derivation of the transport equations for macroscopic variable becomes easier when BGK model replaced the nonlinear collision term of Boltzmann equation by a simpler term. The term is the relaxation of a state of a fluid to equilibrium state. Derivation of numerical schemes (kinetic schemes) is one of the important new applications by BGK model to solve hyperbolic conservation of laws (Bhatnagar, P.L., Gross, E.P and Krook, M., 1954).